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INVESTIGATION OF A SPATIALLY-OSCILLATING VIBRATING SCREEN WITH LONGITUDINAL RIDGES

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Abstract. The aim of the study is to improve the efficiency of classifying fine-grained bulk materials by creating a mathematical model of a vibrating screen with a corrugated screening surface operating in spatial (three-dimensional) vibration mode. The design with an inertial vibration exciter installed under the bottom of the box at an angle to the working surface is considered.

The first part presents a dynamic calculation of the screen. Linear vibrations in the vertical plane of symmetry of the screen and rotational vibrations around the horizontal and vertical axes passing through the center of gravity are considered. Lagrange equations of the second kind are compiled, taking into account inelastic resistance. Amplitude-frequency and phase-frequency characteristics are constructed. In the resonance zone, a sharp increase in the amplitudes of the screen's oscillations is observed. In the operating frequency zone, the amplitudes of the oscillations and the phase characteristics of the system stabilize.

The screen has a corrugated working surface. Therefore, the second part examines the movement of a particle of material within a single corrugation relative to a local coordinate system. Differential equations of motion of the particle along the working surface and perpendicular to the corrugation cross-section are derived. Given the initial conditions, expressions for the displacement of the particle in the specified directions are obtained. Under the action of vibration, the layer of material becomes loose. From this, it can be assumed that the movement of particles approximately characterizes the behavior of the material at individual points on the surface of the screen. Graphical dependencies of the movement of material along the working surface and in the transverse direction were obtained. Their analysis showed a relatively constant movement of particles along the box to the unloading point. Across the width of the screen, there is a significant difference in the speeds of movement in the central part and at the edges. This indicates the presence of intensified mixing of the material in the transverse direction, which has a positive effect on the quality of its classification.

The practical significance lies in the formation of recommendations for selecting the geometry of corrugations, frequency and phase excitation parameters for materials with different particle sizes, as well as in ensuring operation in the resonance range, which reduces peak inertial loads, energy consumption and wear.

The originality and value of the work lie in the construction of a consistent model that, for the first time, systematically combines the geometry of the corrugated surface and the spatial kinematics of the deck with the indicators of the loosening process, creating a basis for multi-criteria optimization of the structural and kinematic parameters of the screen.

Keywords: vibrating screen; spatial vibrations; corrugated screening surface; mathematical modeling; classification efficiency.

1. Introduction

The development of modern screening technologies is aimed at improving the efficiency of classification and separation of bulk materials through new structural and kinematic solutions, optimization of screen geometry, and improvement of the dynamic characteristics of machines. A promising direction is the transition from traditional single-axis vibrations (linear or circular) to multidimensional trajectories and complex spatial modes, which allows intensifying the screening process without increasing the size of the equipment.

Two-shaft self-balancing or self-synchronizing vibration exciters are widely used on screens, which provide rectilinear vibrations of the box at a certain angle to the screening surface. In high-performance screens, the length of the screening surface is 5–8 m, which reduces the thickness of the material layer by 3–10 times. To improve

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the efficiency of classification, screens with variable mesh vibration parameters along the length of the screening surface is proposed [1].

One promising direction is the use of a single-axis inertial vibrator with a vertical axis, which provides spatial vibrations of the working surface. A theoretical analysis of material movement under such conditions was carried out in [2, 3]. The authors took into account the change in vibration viscosity across the thickness of the layer and considered the movement of material along the box and across it within the framework of two separate tasks, which allows describing the complex dynamics of a bulk medium.

Another approach is proposed in [4], where the influence of the geometric parameters of the screen and the location of the vibrator on the movement of material across the working surface is analyzed. The results of the research made it possible to determine the optimal parameters for material movement, and experimental tests [5] confirmed the effectiveness of the proposed design scheme, which ensures simultaneous directed movement of material to the unloading point and its intensive mixing.

The next stage in the improvement of screens was the use of a screening surface with longitudinal corrugations, which contribute to the intensification of material mixing and increase the efficiency of classification [6]. Analytical studies of this design solution are not yet available.

The problem of taking dissipative processes into account in vibrating systems is of fundamental importance for an adequate description of their dynamics. The classic work [7] sets out the basics of the theory of internal friction in the oscillations of elastic systems, the mechanisms of energy losses, and methods of taking damping into account in analytical models.

This direction has been developed by domestic researchers in their works devoted to mathematical modeling of real structures. Thus, in [8], a mathematical model of a vibrating separator with a sequential arrangement of screens was constructed, which made it possible to evaluate the influence of vibration parameters and geometry on the efficiency of separation of finely dispersed materials. In [9], the dynamics of vibrating machines for processing product surfaces were modeled, determining the patterns of oscillatory motion and the influence of design and operating parameters on the processing process. In the study [10], a model of a two-mass oscillatory system with parallel elastic and damping elements was proposed, which allows taking into account both elastic and dissipative properties of the system for calculations and optimization of vibration drives and working bodies.

The works of Professors Uchitel and Zaseselsky [11, 12], devoted to the spatial oscillations of the working bodies of vibrating machines, deserve special attention. In [11], the trajectory field of a vibration exciter with inhomogeneous and spatial vibrations during the sorting of metallurgical charge is determined. The interaction of vertical and horizontal vibration components and their influence on the movement of material along and across the surface is considered. Experimental confirmation is given in [12], where the mode of impulse vibrations ("beating") is investigated, which provides more intensive mixing of the material, reduces stagnation on the surface, and improves the uniformity of classification.

Thus, the works of Uchitel and Zaseselsky form the theoretical and experimental basis for the study of spatial vibrations and can serve as a guideline for the design and optimization of vibratory classifiers.

Vibrating screens are used to classify fine-grained materials, but this process is labor-intensive and relatively inefficient. For such material, the traditional approach to layer thickness, which was supposed to be 3–4 average particle sizes, is no longer followed; in order to increase productivity, it is fed onto the screening surface in a relatively thick layer. At the same time, the live cross-section of the screening surface is relatively small. Under these conditions, separation by size occurs as a result of interrelated processes of segregation, screening, and vibratory transport. The intensity of these processes is determined by the particle size distribution, particle shape, physical and mechanical properties of the raw material, and the thickness of the layer, which decreases during transportation. With this in mind, it is necessary to make a rational choice of the structural and dynamic parameters of the screen.

It is common practice to install two-shaft self-balancing or self-synchronizing vibration exciters on screens, which form rectilinear vibrations of the box at a certain angle to the screening surface, as proposed by Professor V.P. Nadutyi. In high-performance screens, the length of the screening surface reaches 5–8 m; during the screening process, the thickness of the material layer can decrease by 3–10 times. To increase the efficiency of classification, screens with variable parameters of sieve vibrations along the screening surface is proposed.

A promising direction for the classification of finely dispersed materials is the use of a single-axis inertial vibration exciter, the axis of which is located in a vertical plane and provides spatial vibrations of the working surface. The theoretical justification for such material movement was provided by Professor L. M. Tishchenko, who considered a loose medium as a system that acquires the properties of a liquid under the action of vibration, taking into account the change in vibration viscosity across the thickness of the layer. The complex movement of material along and across the box was studied in two separate settings.

An alternative approach to solving this problem was proposed by Professor V. P. Franchuk. In his works [13–15] and the research of his school, the influence of the geometric parameters of the screen and the location of the vibration exciter on the kinematics of the working deck and the trajectory of particle movement was analyzed. The results obtained made it possible to substantiate the rational ranges of structural and kinematic parameters that ensure favorable conditions for material transport across the screening surface. Experimental verification confirmed the effectiveness of the proposed design scheme: along with the controlled directional movement of material to the unloading zone, intensive continuous mixing of the layer is ensured.

The next stage in the improvement of screens was the use of a screening surface with longitudinal corrugations, which intensify the mixing of material and increase the efficiency of classification. Professor V. P. Franchuk and his research team received a patent for a utility model for this development. At the same time, this design solution has so far remained outside the scope of systematic analytical consideration.

Corrugations are longitudinal ridges on the screen that form a periodic relief. Due to the change in the local geometry of the surface, the vectors of contact forces and friction forces vary; as a result, the particles not only slide, but also undergo short-term overturning over the ribs and tossing. Such micro-modes destroy the crust of clumped material, activate size segregation processes (small particles move downwards, large ones move to the upper layers), and prevent flow compaction.

The screening surface vibrates simultaneously in three directions (***x***, ***y***, ***z***). Correctly selected amplitude and phase ratios create time intervals when the normal pressure on the screen decreases, the layer loosens, and the relative velocity of small particles relative to the holes increases. During these time «windows», the probability of particles passing through without additional energy expenditure increases significantly.

The scientific problem is to develop a consistent mathematical model of a vibrating screen with a sieving surface equipped with longitudinal corrugations, which operates in the mode of spatial (three-dimensional) vibrations.

The purpose of the model is to formally and quantitatively describe the influence of the geometric parameters of the corrugations (height, pitch, profile), amplitude-frequency and phase characteristics of the vibrations, as well as the physical and mechanical properties of the material on the kinematics of particle movement, the efficiency indicators of the classification process, and the specific energy consumption.

2. Methods

The simplest design of such a screen (Fig. 1) includes: *1* – a box as a rigid oscillating body with a screening surface; *2* – elastic shock absorbers connecting the box to the base; *3* – inertial vibration exciter located in a vertical plane at an angle β to the working surface; *4* – material loading device; *5* – unloading device.

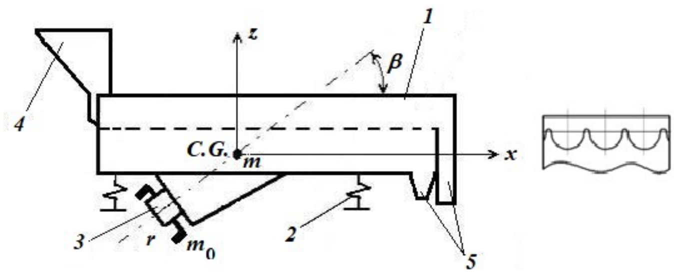


Figure 1 – Structural diagram of the screen

A special feature of the design is the location of the vibrator so that the projection of the exciting force onto the vertical plane passes through the center of mass of the oscillating system. This eliminates the tendency for uneven motion and ensures a stable spatial vibration mode.

The screening surface is made in the form of longitudinal cylindrical perforated streams (ribs), on which the material to be screened is transported and distributed.

Due to the spatial movement of the working surface, the material performs directed movements along the *x*-axis and circular movements in the transverse direction. When screening fine-grained materials, this ensures improved contact of the particles with the perforated surface and increases the productivity and efficiency of the process.

The dependence of the dynamic characteristics of the screen and the parameters of material movement on the size of the screening surface and the location of the drive is considered taking into account the following assumptions:

- The mass of the screen box is proportional to its length and width, and the moment of inertia takes into account the end elements of the structure (loading and unloading devices).
- The attachment of the drive shifts the center of gravity of the screen by an amount proportional to the mass of the drive and the distance from its center to the center of mass of the box.
- When the drive is shifted, the condition is satisfied that the projection of the disturbing force vector onto the vertical plane passes through the center of gravity of the box.

The dynamic calculation diagram with the designation of geometric and dynamic parameters is shown in Fig. 2.

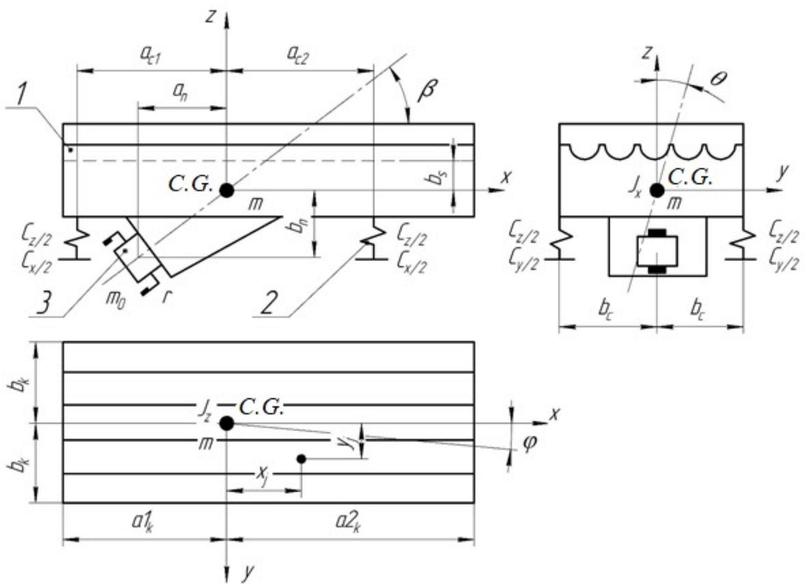


Figure 2 – Calculation dynamic diagram of the screen

Fig. 2 shows the calculation-dynamic diagram of a vibrating screen with a screening surface equipped with longitudinal corrugations in three projections (longitudinal *xOz*, transverse *yOz*, and plan *xOz*). The oscillating part of the installation is represented by a box with a center of mass fixed on elastic elements. Spatial oscillations are driven by a twin-shaft inertial vibrator located in a vertical plane at an angle to the working surface.

The diagram shows: m – mass of the moving parts of the screen; m_0 – mass of the drive unbalanced masses; r – eccentricity of the center of gravity of the unbalanced masses; J_x, J_z — moments of inertia of the box around the x and z axes relative to the center of mass; C_x, C_y – stiffness of elastic connections in the direction of the x, y, and z axes; β – angle of inclination of the working surface (technological angle that sets the direction of material transportation); ϕ – angle of rotation around the x axis; θ – angle of rotation around the z-axis; b_n – distance from the center of mass of the box to the axis of the unbalanced shaft along the x-axis; a_n – distance from the center of mass of the box to the axis of the unbalanced shaft along the z-axis; a_0 – distance from the center of the unbalanced masses to the center of gravity of the drive; a_{c1}, a_{c2} – distance from the elastic elements to the center of mass of the box along the length of the screen; b_c – distance from the elastic elements to the center of mass of the box along the width.

The coordinate system is chosen so that the x-axis is directed along the working surface, the z-axis is directed vertically upward, and the y-axis is directed transversely.

3. Theoretical part

This diagram is used to compile equations of motion according to Lagrange II, in which kinetic and potential energies are determined by the given parameters, we obtain a system of equations describing the motion of the screen in the form:

$$\begin{cases} M\ddot{q}_1 + \left(C_x \sin\beta + C_z \cos\beta\right) q_1 = m_0 r\omega^2 \sin\left(\omega t\right) \\ M\ddot{y}_1 - m_0 a_n \ddot{\phi} - m_0 b_n \ddot{\theta} + C_y y = m_0 r\omega^2 \cos\left(\omega t\right) \\ I_{x1}\ddot{\phi} - m_0 a_n \ddot{y}_1 - m_0 a_n b_n \ddot{\theta} + 0{,}5\, C_y \left(a_{c1}^2 + a_{c2}^2\right)\phi = -m_0 r\omega^2 a_n \cos\left(\omega t\right) \\ I_{z1}\ddot{\theta} - m_0 b_n \ddot{y}_1 - m_0 a_n b_n \ddot{\phi} + C_x b_c^2 \theta = -m_0 r\omega^2 b_n \cos\left(\omega t\right) \end{cases} \quad (1)$$

In the system of equations (1), it is customary to

$$M = m + m_0;\quad I_{x1} = I_x + a_n^2 m_0;\quad I_{y1} = I_y + b_n^2 m_0. \quad (2)$$

The first equation of system (1) is independent, so it can be solved separately. It is convenient to present the solution of this equation in the form:

$$q_1 = A\sin\left(\omega t\right), \quad (3)$$

where $A = \dfrac{m_0 r\omega^2}{\left(C_x \sin\beta + C_z \cos\beta\right) - M\omega^2}$.

The solution of the remaining three equations of system (1) according to the method of undetermined coefficients should be presented in the form

$$y = Y\cos\omega t, \quad \varphi = \Phi\cos\omega t, \quad \theta = \Theta\cos\omega t. \tag{4}$$

The amplitude values of linear and angular displacements in the elastic formulation of the problem are as follows:

$$\begin{cases} Y = m_0 r\omega^2 \begin{Bmatrix} \left[0{,}5\,C_y\left(a_{c1}^2 + a_{c2}^2\right) - J_{x1}\omega^2\right]\left(C_x b_c^2 - J_{z1}\omega^2\right) - \\ -3\left(m_0 a_n b_n \omega^2\right)^2 + \\ + m_0\omega^2\left[a_n^2\left(C_x b_c^2 - J_{z1}\omega^2\right) + b_n^2\left(0{,}5\,C_y\left(a_{c1}^2 + b_{c1}^2\right) - J_{x1}\omega^2\right)\right] \end{Bmatrix} * Z_n^{-1}, \\ \Phi = m_0 r\omega^2 \begin{Bmatrix} m_0 a_n b_n^2 \omega^4 - a_n\left(C_y - M\omega^2\right)\left(C_x b_c^2 - J_{z1}\omega^2\right) - \\ - m_0 a_n \omega^2\left[\left(C_x b_c^2 - J_{z1}\omega^2\right) - b_n^2\left(C_y - M\omega^2\right)\right] \end{Bmatrix} * Z_n^{-1}, \\ \Theta = m_0 r\omega^2 \begin{Bmatrix} m_0 a_n^2 b_n \omega^4 - b_n\left(C_y - M\omega^2\right)\left(0{,}5\,C_y\left(a_{c1}^2 + a_{c2}^2\right) - J_{z1}\omega^2\right) - \\ - m_0 b_n \omega^2\left[\left(0{,}5\,C_y\left(a_{c1}^2 + a_{c2}^2\right) - J_{z1}\omega^2\right) - a_n^2\left(C_y - M\omega^2\right)\right] \end{Bmatrix} * Z_n^{-1}. \end{cases} \tag{5}$$

The denominator in expression (5) is determined by the dependence

$$\begin{aligned} Z_n = {} & \left(C_y - M\omega^2\right)\left\{\left[0{,}5\,C_y\left(a_{c1}^2 + a_{c2}^2\right) - J_{x1}\omega^2\right]\left(C_x b_c^2 - J_{z1}\omega^2\right) - \left(m_0 a_n b_n \omega^2\right)\right\} - \\ & - \left(C_x b_c^2 - J_{z1}\omega^2\right)\left(m_0 a_n \omega^2\right)^2 - \left[0{,}5\,C_y\left(a_{c1}^2 + a_{c2}^2\right) - J_{x1}\omega^2\right]\left(m_0 b_n \omega^2\right)^2 + \\ & + 2m_0^3\left(a_n b_n\right)^2\omega^5. \end{aligned} \tag{6}$$

In accordance with the Bokka–Schlipe hypothesis, inelastic supports are considered to generate forces proportional to the first power of the deformation velocity of elastic connections and independent of the deformation frequency. This approach is consistent with the concept of complex elastic modulus introduced by E. S. Sorokin. For this formulation, the equivalent complex stiffnesses are as follows

$$\overline{C}_x = C_x\left(1 + i\lambda_c\right), \quad \overline{C}_y = C_y\left(1 + i\lambda_c\right), \quad \overline{C}_z = C_z\left(1 + i\lambda_c\right) \tag{7}$$

where *i* is an imaginary unit, and λ_c *is* the coefficient of internal friction (losses).

The coefficient λ_c is related to the energy absorption coefficient ψ and the damping coefficient δ by the following relationship

$$2\pi\lambda_c = \psi = 2\delta. \tag{8}$$

Substituting the complex stiffnesses (7) into expressions (3), (5), and (6), we obtain the complex amplitudes of the longitudinal, transverse, and angular components of the oscillations. The actual, physically measurable amplitudes and phase shifts are determined as the modules and arguments of the corresponding complex quantities.

$$A = \left|\overline{A}\right|, Y = \left|\overline{Y}\right|, \Phi = \left|\overline{\Phi}\right|, \Theta = \left|\overline{\Theta}\right|,$$
$$\alpha A = arg\left(\overline{A}\right), \alpha Y = arg\left(\overline{Y}\right), \alpha\Phi = arg\left(\overline{\Phi}\right), \alpha\Theta = arg\left(\overline{\Theta}\right). \quad (9)$$

In this case, the displacements in the coordinates take the form

$$q_1 = A\sin\left(\omega t + \alpha A\right), \quad y = Y\cos\left(\omega t + \alpha Y\right),$$
$$\phi = \Phi\cos\left(\omega t + \alpha\Phi\right), \ \theta = \sin\left(\omega t + \alpha\Theta\right) \quad (10)$$

Fig. 3a shows the amplitude-frequency characteristics of translational (linear) and angular (rotational) vibrations of the vibrating system of the screen. In the vicinity of the natural frequencies (resonance zone), a sharp increase in amplitudes is observed. In the operating frequency range close to $\omega \approx 100$ rad/s, the amplitudes acquire stationary (quasi-steady) values, and the phase characteristics of the system stabilize, as illustrated in Fig. 3b. This creates the prerequisites for a reproducible and controllable mode of operation of the screen.

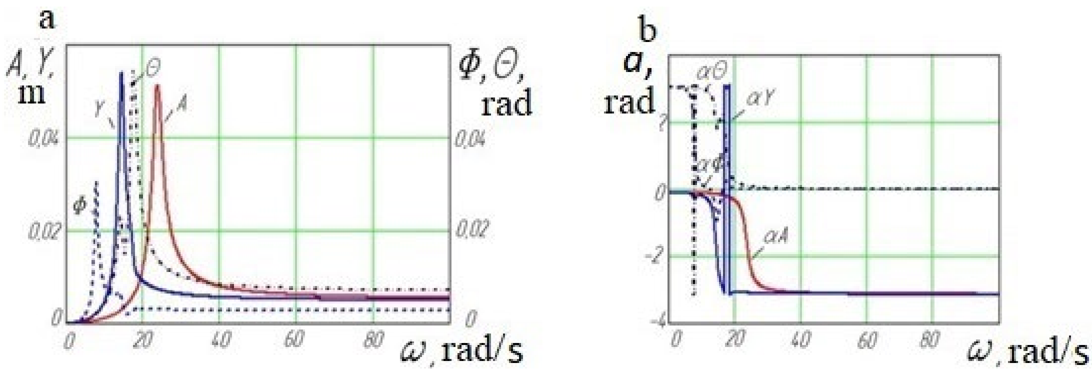


Figure 3 – Amplitude-frequency and phase-frequency characteristics of the vibrating screen

Let us consider the movement of material on the surface of a screen that has longitudinal corrugations of a cylindrical shape in cross section (Figs. 2, 4). In this case, we assume that the screen operates in working mode with a frequency of $\omega = 100$ rad/s.

The movement of the screen surface at point C in the x, y, and z coordinates, taking into account the decomposition q_1 horizontally and vertically, as well as the movement in the y, φ, and θ coordinates, can be written as (the x_c coordinate is normal to the yoz plane and is not shown in the figure)

$$x_c = q_1 \cos\beta + (Y_0 + Y_c)\varphi,$$
$$y_c = y + X_c\varphi + (Z_0 + Z_c)\theta,$$
$$z_c = q_1 \sin\beta + (Y_0 + Y_c)\theta.$$

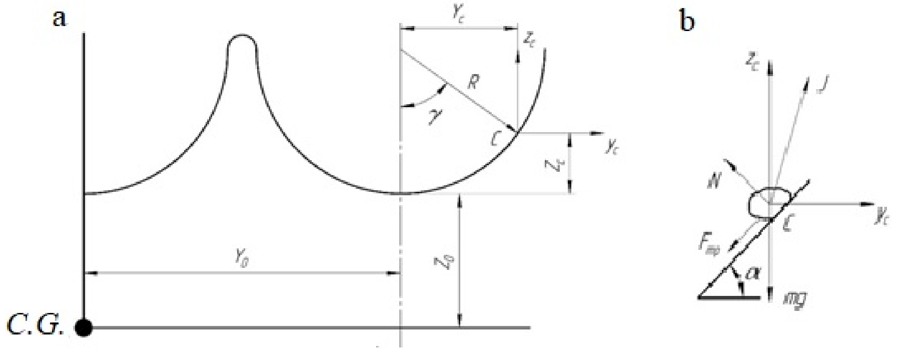


Figure 4 – Calculation diagram for determining the movement of material on the surface of the screen

Substituting the values *of* q_1 , y, φ, and θ from (3) and (4), taking into account (5), (6), (7), and (9), and decomposing the oscillations into harmonics, we obtain the displacement at point *C* as:

$$x_c = A_{xc} \sin(\omega t + \vartheta_{xc}),$$
$$y_c = A_{yc} \cos(\omega t + \vartheta_{yc}), \qquad (11)$$
$$z_c = A_{zc} \cos(\omega t + \vartheta_{zc}),$$

where the expressions for the coordinates x, y, z, amplitude A_c and phase shift ϑ_c are determined by the dependencies on the moduli and arguments of complex numbers (9).

Under the action of vibrations, the material moves away from the working surface along the screen to the unloading point (along the *x*-axis) and in the transverse direction (in the *yoz* plane, Fig. 4).

Let us denote:

- ξ_{mx} – movement of material along the *x-axis*;
- ξ_{mp} , ,ξ_{mn} – movement of material along and normal to the screen surface at point *C*;
- ξ_{cp} , ,ξ_{cn} – movement of the screen along and normal to the surface at point *C*;
- α_1 – angle of inclination of the screen surface towards the discharge side;
- α – angle of inclination of the screen surface at point *C* in the *yoz* plane (in this case, $\alpha = \gamma$),

Then the equation of motion can be written as

$$m\,\ddot{\xi}_{mx} = m\,\ddot{x}_c + m\,g\sin\alpha_1 - F_{mp},$$
$$m\,\ddot{\xi}_{mp} = m\,\ddot{\xi}_{cp} - m\,g\sin\alpha - F_{mp}, \qquad (12)$$
$$m\,\ddot{\xi}_{mn} = m\,\ddot{\xi}_{cn} - m\,g\cos\alpha + N.$$

The motion along the generatrix of the working surface at point C is determined as follows:

$$\xi_{cp} = y_c\cos\alpha + z_c\sin\alpha = A_{cp}\sin\left(\omega t + \vartheta_{cp}\right), \qquad (13)$$

The motion along the normal to this surface is determined as follows:

$$\xi_{cn} = y_c\sin\alpha + z_c\cos\alpha = A_{cn}\sin\left(\omega t + \vartheta_{cn}\right), \qquad (14)$$

where the amplitudes of oscillations A_{cp}, A_{cn} and phase angles $\vartheta_{cp}, \vartheta_{cn}$ are determined based on the dependencies for coordinates y and z, given in the second and third equations (11), using expressions for the moduli and arguments of complex numbers (9).

The parameters of material motion are determined from equations (12) under the following initial conditions:

$$\xi_{mx} = \xi_{mp} = \xi_{mn} = 0,\ \dot{\xi}_{mx} = \dot{\xi}_{mp} = \dot{\xi}_{mn} = 0,\ N = 0,\ F_{mp} = 0 \text{ at } t = t_0 \qquad (17)$$

Taking into account that at the moment of separation of the material from the surface, the noise $\ddot{\xi} = 0$ from the third equation (12) after substitution (14), we obtain

$$\sin(\omega t_0 + \vartheta_{cn}) = \frac{g\cdot\cos\alpha}{\omega^2 A_{cn}}, \text{ from which, } \psi_0 = \omega t_0 = \arcsin\left(\frac{1}{\Gamma_c}\right) - \vartheta_{cn} \qquad (18)$$

where $\Gamma_c = \dfrac{A_{cn}\cdot\omega^2}{g\cdot\cos\alpha}$ is the vibration transport mode coefficient.

Integrating the third equation (14) and taking into account that after flight the material contacts with the surface of the screen ($\xi_{mn} = 0$) at the moment of time t_B ($\psi_B = \omega t_B$), we obtain the equation for determining the time phase angle of encounter

$$\sin(\psi_0 + \vartheta_{cn}) - \sin(\psi_B + \vartheta_{cn}) - \frac{1}{2}\frac{(\psi_B - \psi_0)^2}{\Gamma_c} + \qquad (19)$$
$$+(\psi_B - \psi_0)\cos(\psi_0 + \vartheta_{cn}) = 0$$

Further integrating the first and second equations of system (12) under the initial conditions (17), we obtain expressions for the displacement of material along the longitudinal axis of the screen and along the stream generatrix.

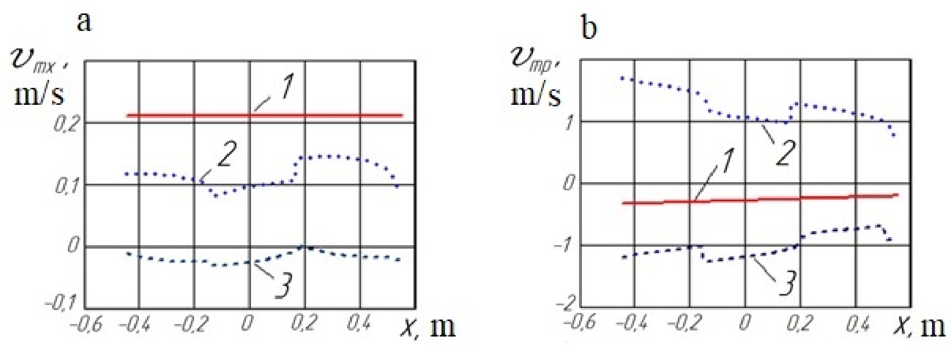
$$\xi_{mx} = A_{xc}\left[\begin{aligned}&\sin(\psi_0+\vartheta_{xc})-\sin(\psi_B+\vartheta_{xc})+\frac{1}{2}\frac{g(\psi_B-\psi_0)^2}{A_{xc}\omega^2}\sin(\alpha_1)+\\&+(\psi_B-\psi_0)\cos(\psi_0+\vartheta_{xc})\end{aligned}\right] \quad (20)$$

$$\xi_{mp} = A_{cp}\left[\sin(\psi_B+\vartheta_{cp})-\sin(\psi_0+\vartheta_{cp})+\frac{1}{2}\frac{(\psi_B-\psi_0)^2}{\Gamma_c}tg(-\alpha)-(\psi_B-\psi_0)\cos(\psi_0+\vartheta_{cp})\right]$$

After substituting $\Delta\xi_{mx}$ and $\Delta\xi_{mp}$ into the above equations, the average velocity of material displacement is determined as

$$\begin{aligned}v_{mx} &= \frac{\omega}{2\pi p}A_{xc}[\sin(\psi_0+\vartheta_{xc})-\sin(\psi_B+\vartheta_{xc})+\frac{1}{2}\frac{g(\psi_B-\psi_0)^2}{A_{xc}\omega^2}\sin\alpha_1+\\&+(\psi_B-\psi_0)\cos(\psi_0+\vartheta_{xc})]\\ v_{mp} &= \frac{\omega}{2\pi p}A_{cp}[\sin(\psi_0+\vartheta_{cp})-\sin(\psi_B+\vartheta_{cp})+\frac{1}{2}\frac{(\psi_B-\psi_0)^2}{\Gamma_c}tg(-\alpha_1)-\\&-(\psi_B-\psi_0)\cos(\psi_0+\vartheta_{cp})]\end{aligned} \quad (21)$$

Fig. 5 shows graphs of velocity in the longitudinal (Fig. 5a) and transverse (Fig. 5b) directions depending on the distance to the center of gravity of the screen along the x-axis.



1 – in the central part of the stream; 2, 3 – near the edge zones of the flow

a – longitudinal component of the average velocity v_{mx} depending on the longitudinal coordinate X,
b – transverse component of the average velocity v_{mp} depending on the longitudinal coordinate X

Figure 5 – Distribution of average material movement speeds along the deck

As it can be seen from Fig. 6, in the longitudinal direction, the material in the total mass moves uniformly in the direction of unloading, slightly decreasing towards the edge of the flow. In the transverse direction, there is a significant difference in the speeds of material movement in the central part and at the edges of the flow, which contributes to its intensive mixing and classification process.

The material flow along the *x*-axis reaches a speed of 0.2 m/s in the direction of unloading (Fig. 6a). Movement in the *YOZ* plane is characterized by a certain degree of chaos with a change in the sign of velocity, which also contributes to the intensive mixing of the material (Fig. 6b).

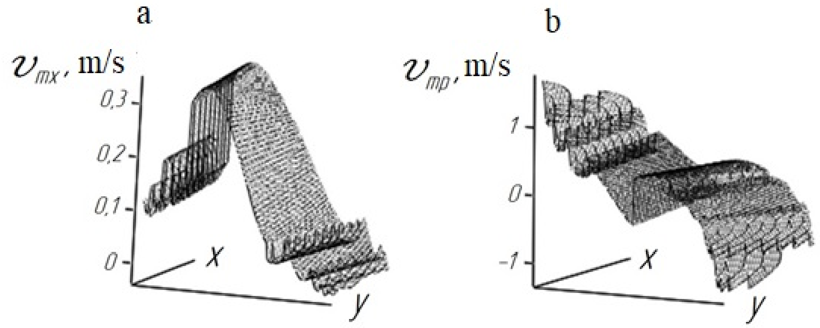


Figure 6 – The speed of material movement in the direction of discharge (a) and in the direction normal to the *x*-axis (b)

4. Results and discussion

Thus, the existence of a range with «flat» amplitude-frequency and phase-frequency characteristics (Fig. 3a, b), in which the modules of the derivatives $|\partial A/\partial \omega|$ and $|\partial \varphi/\partial \omega|$ acquire small values. Operation in this zone, located outside the resonance ω, ensures:

- stability of the amplitudes of the translational and angular components of motion and reproducibility of the interaxial phase shifts *x, y, z*;
- the absence of sharp amplitude jumps («galloping») with moderate variations in stiffness and damping;
- controllability of the trajectories of representative points of the screening surface and cyclical action on the ball;
- energy stability of the mode — reduction of peak inertial loads and «parasitic» losses, which increases the service life of the components.

For the prototype under study, the working zone is concentrated near $\omega \approx 100$ rad/s with robustness to parameter changes of ±(5–10)%.

The criterion for selecting the operating frequency can be formulated as $\partial A/\partial \omega < \varepsilon_A$ and $\partial \varphi/\partial \omega < \varepsilon_\varphi$ while simultaneously meeting the technological requirements for acceleration in the *z* direction.

The estimates are based on frequency expressions (5)–(6), (9)–(10) with equivalent consideration of damping (7)–(8).

An analytical condition for the loss of contact between a particle and the surface $N(t) = 0$ (formula (18)) is derived, equivalent to the criterion $\Gamma(t) = a_z(t)/g > 1$ (formula (21)).

It is shown that:

• the duration of «windows» τ_w, within which the normal pressure decreases and the layer loosens, increases with an increase in the vertical amplitude A_z and is determined by phase differences $\Delta\varphi_{xz}$, $\Delta\varphi_{yz}$;

• optimal phase adjustment synchronizes the moments of minimum pressure with peak planar velocities, thereby increasing the probability of the fine fraction passing through the holes per cycle;

• the periodicity of the «windows» is determined by the fundamental excitation frequency, which allows phase control of the process intensity without additional energy input;

• for wet/cohesive materials, the fulfillment of the separation condition requires larger A_z or larger Δ_φ, which corresponds to the physical need for deeper decompression of the layer;

• an excessive increase in A_z transfers individual particles into a «ballistic» mode with the risk of transfer to failure zones; therefore, $\tau\omega/T = \xi$ (the fraction of the «window» time in the period) has an optimum that should be determined for a given granulometry and moisture content.

The mathematical apparatus is based on the parameterization of surface point trajectories (11)–(13), particle motion equations (14)–(16) with initial conditions (21), and re-contact equations (23).

Taken together, the results confirm that the corrugated screening surface, in combination with spatial oscillations, forms a controlled field of velocities and pressures in the layer, increasing stratification, reducing the of clogging of the holes, and increasing the efficiency of separation without a significant increase in energy consumption. The obtained criteria and indicators (ξ_w, ,λ) form the basis for parametric optimization of the design and operating modes of the screen.

The analytical dependencies are consistent with the constructed amplitude-frequency (AF) and phase-frequency (PF) characteristics (Fig. 3): in the working frequency range $\omega \approx 100$ rad/s, the amplitudes and phase angles acquire stationary values. This prevents the appearance of «galloping» and ensures a reproducible 3D mode at constant energy consumption.

The consistency of analytics and numerical calculations indicates that the adopted linearized damping model adequately describes the attenuation of oscillations in the studied range of parameters.

The results of the calculations (Figs. 5–6) made it possible to reconstruct the average and instantaneous particle transport velocities:

- along the x-axis, there is predominantly uniform transport in the direction of unloading (about 0.2 m/s), with a slight decrease near the side edges of the corrugated streams; this gradient is associated with local energy losses at contact interactions and partial accumulation of material in coastal areas;
- in the transverse plane yOz, a variable velocity regime (locally quasi-chaotic) is recorded with the formation of smaller recirculation cells between the corrugations. Transverse oscillatory components intensify layer mixing and promote stratification:

the fine fraction sinks faster to the working openings, while the coarse fraction is carried to the upper layers.

The combination of longitudinal corrugations with 3D kinematics creates temporal and spatial «windows» of loosening: at moments of reduced normal pressure on the screen, the relative speed of fine particles to the openings increases, increasing the probability of their passage without a noticeable increase in energy consumption. At the same time, the corrugations destroy conglomerates of clumped material, reduce the tendency to form «crusts» and slow down the process of clogging the holes.

The qualitative sensitivity of the indicators to the corrugation parameters is consistent with the physical interpretation:

- an increase in height h_r enhances micro-throwing and transverse shearing, improving mixing, but an excessive increase in h_r can prolong the residence time of particles on the screen and increase local losses;
- reducing the pitch p_r increases the frequency of contact with the ribs and accelerates stratification, but too small p_r increases the risk of mechanical «locking» of fractions and local clogging;
- the radii of curvature of the ribs reduce impact loads and wear, with almost no deterioration in the mixing effect.

The most noticeable is the sensitivity to the vertical amplitude A_z and phase shifts between the vertical and planar components: they determine the duration of the loosening «windows» and the depth of layer decompression. In the operating range of the non-resonant frequency, phase stabilization minimizes flow fluctuations and ensures the repeatability of particle trajectories.

The stationarity of amplitudes in the operating frequency range limits peak inertial loads on load-bearing elements and reduces «parasitic» losses, which has a positive effect on the specific energy consumption of the process. The generalized vibration transport coefficient λ, estimated from the obtained dependencies, correlates with the classification efficiency: an increase in λ is accompanied by an increase in the probability of fine fraction passage and a decrease in hole clogging.

The conclusions were made based on assumptions about the linear characteristics of the supports and equivalent viscous damping; the model does not take into account possible large elastic deformations of the screen panels and the explicit dependence of contact parameters on humidity and temperature. Further research should focus on experimental validation in an extended range of humidity and granulometry, coupling with an elastic-deformable model of panels, and multi-criteria optimization of parameters $\{h_r, p_r, A_x, A_y, A_z, \phi_x, \phi_y, \phi_z, \beta\}$ taking into account restrictions on wear and energy consumption.

Thus, the results obtained confirm that the corrugated screening surface, in combination with spatial vibrations, forms a controlled field of velocities and pressures in the layer, increases stratification, reduces clogging of holes, and increases separation efficiency without additional energy consumption. This approach provides a scientifically sound basis for further engineering optimization and scaling of designs.

5. Conclusions

1. The proposed 3D kinematic diagram of a screen with a corrugated surface and the corresponding mathematical model (1)–(23) provide a quantitative description of the transport, loosening, and stratification of finely dispersed materials.

2. Outside the resonance range, stabilization of the amplitudes and phases of oscillations is achieved, which is a necessary prerequisite for controlled separation.

3. Corrugations significantly change the local kinematics of particles (sliding, overturning, tossing), enhance mixing, and reduce layer compaction.

4. Criteria are formulated and expressions are obtained for the average transport velocities and the vibration transport coefficient λ, which are suitable for engineering optimization of the process.

5. The results obtained serve as a theoretical basis for selecting the geometry of corrugations and parameters of 3D vibrations, taking into account the properties of the material (moisture content, granulometry, cohesion) and the requirements for the productivity and energy intensity of the process.

Conflict of interest

Authors state no conflict of interest.

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ДОСЛІДЖЕННЯ ПРОСТОРОВО-КОЛИВНОГО ВІБРАЦІЙНОГО ГРОХОТА З ПОЗДОВЖНІМИ РИФЛЕННЯМИ

Франчук В., Заболотний К., Анциферов О., Гавриленко С.

Анотація. Мета дослідження – підвищити ефективність класифікації дрібнодисперсних сипких матеріалів шляхом створення математичної моделі вібраційного грохота з рифленою просіювальною поверхнею, який працює в режимі просторових (тривимірних) коливань. Розглядається конструкція з інерційним віброзбуджувачем, установленим під днищем короба під кутом до робочої поверхні.

У першій частині проведено динамічний розрахунок грохоту. Розглядаються лінійні коливання у вертикальній площині симетрії грохоту та поворотні коливання навколо горизонтальної і вертикальної осей, що проходять через центр тяжіння. Складено рівняння Лагранжа II роду з урахування непружних опорів. Побудовано амплітудно-частотні і фазо-частотні характеристики. У зоні резонансу спостерігається різке зростання амплітуд коливань грохоту. У зоні робочих частот амплітуди коливань і фазні характеристики системи стабілізуються.

Грохот має рифлену робочу поверхню. Тому у другій частині досліджується рух частинки матеріалу в межах однієї рифлі відносно локальної системи координат. Складені диференційні рівняння руху частинки вподовж робочої поверхні і перпендикулярно по перерізу рифлі. З них при заданих начальних умовах отримані вирази для переміщення частинки в заданих напрямках. Під дією вібрації шар матеріалу розпушується. Звідси можна вважати, що рух частинки наближено характеризує поведінку матеріалу. в окремих точках поверхні грохота. Отримані графічні залежності руху матеріалу вподовж робочої поверхні і в поперечному напрямку. Їх аналіз показав відносно сталий рух частинок вподовж короба до точки розвантаження. За шириною грохота спостерігається значна різниця у швидкостях руху в центральній частині і біля країв. Це свідчить про наявність інтенсифікації перемішування матеріалу у поперечному напрямку, що позитивно впливає на якість його класифікації.

Практична значущість полягає у формуванні рекомендацій щодо вибору геометрії рифлень, частотних та амплітудно-фазових параметрів збудження для різних за гранулометрією та вологістю матеріалів, а також у забезпеченні роботи в позарезонансній області, що знижує пікові інерційні навантаження, енергоємність і знос.

Оригінальність і цінність роботи полягають у побудові узгодженої моделі, яка вперше системно поєднує геометрію рифленої поверхні та просторову кінематику деки з показниками процесу розпушення, створюючи підґрунтя для багатокритеріальної оптимізації конструктивно-кінематичних параметрів грохота.

Ключові слова: вібраційний грохот; просторові коливання; рифлена просіювальна поверхня; математичне моделювання; ефективність класифікації.